Curvas parametrizadas - trajetórias

Exemplo genérico 2D

```
> restart; with(plots) :
```

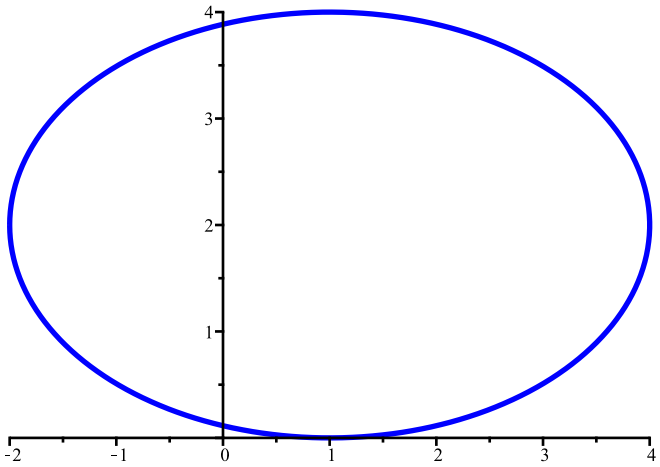
Digamos que temos a parametrização de uma curva no plano. Em coordenadas cartesianas, basta termos x(t) e y(t) funções reais a um parâmetro t .

A sintaxe para se desenhar uma curva parametrizada no plano é:

plot([x(t), y(t), t= a..b], opçoes);

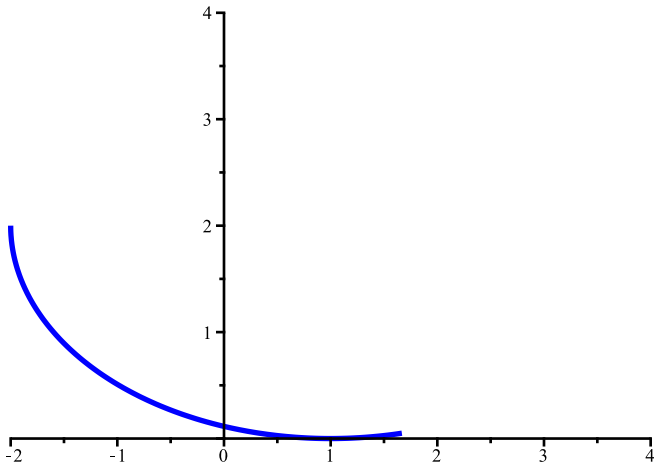
Vamos exemplificar com uma elipse:

```
> plot([1 + 3*cos(t), 2 + 2*sin(t), t = -Pi..Pi], color = blue, thickness = 2, scaling
      = constrained); # o desenho pronto e estático
```



criar uma sequência de quadros a serem mostrados a uma velocidade suficiente para dar a idéia de movimento:

```
> N := 7 : #(8=N+1 será o número de quadros)#
  for i from 0 to N do
  s := i/N;
  T := -Pi + (t + Pi) * s :
  quadro[i] := plot([1 + 3*cos(T), 2 + 2*sin(T), t = -Pi..Pi], color = blue, thickness = 2) :
  od:
> display([seq(quadro[i], i = 0..N)], insequence = true, scaling = constrained);
```



Uma ilustração da parametrização utilizada para Geometria Analítica

No caso da parametrização da elipse dada acima, o parâmetro t tem um significado geométrico, que pode ser visualizado através da seguinte animação:

A programação da animação:

```
> restart; with(plots) : with(linalg) :
  with(plottools) : # somente do Release 4 em diante
```

```
> CE := circle( [1, 2], 3, color = blue) :
  CI := circle([1, 2], 2, color = violet) :
  elipse := plot( [1 + 3 * cos(t), 2 + 2 * sin(t), t = 0 ..2 * Pi], color = red, thickness = 3) :
  eixos := plot( {[[-2, 2], [4, 2]], [[1, -1], [1, 5]]}, color = orange) :
  theta := Pi / 6 : n := 20 :
  for i from 0 to n do
  raio := plot( [[1, 2], [1 + 3 * cos(theta), 2 + 3 * sin(the • ta)]], color = black, thickness = 2) :
  x0 := 1 + 3 * cos(theta) : y0 := 2 + 2 * sin(theta) :
  ponto := circle([x0, y0], .1, color = red) :
  rx := plot([[x0, 2 + 3 * sin(theta)], [x0, 2]], color = black) :
  ry := plot([[1 + 3 * cos(theta), y0], [1, y0]], color = black) :
  theta := theta + 2 * Pi / n :
  quadro[i] := display( {CE, CI, elipse, raio, rx, ry, eixos, ponto}, scaling = constrained) :
  od:
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```
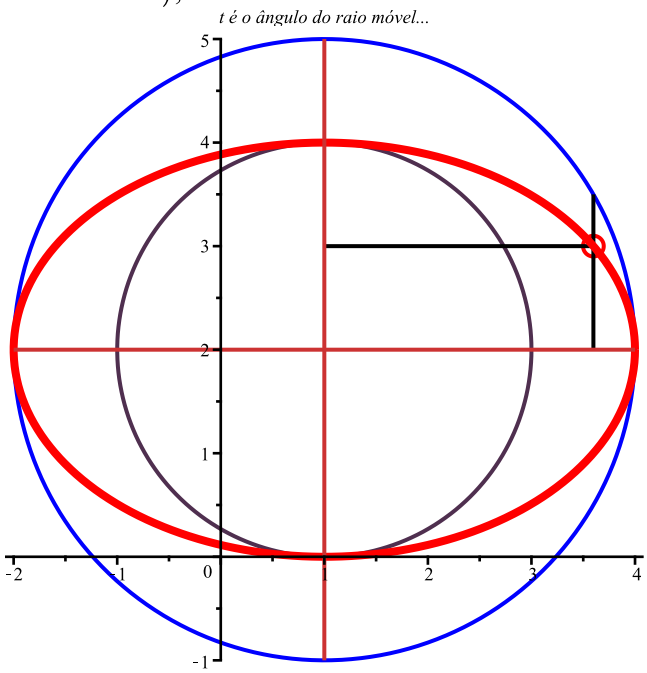

```
> display( [seq(quadro[ j ], j = 0 ..n ) ], scaling = constrained, insequence = true, title
      = `t é o ângulo do raio móvel... `);
```



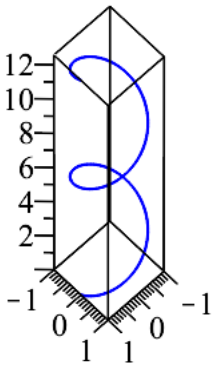
Exemplo genérico 3D -(curva parametrizada)

Neste caso, a sintaxe para plotar uma curva em 3D tem ligeira diferença em relação ao caso 2D:

spacecurve([x(t), y(t), z(t)], t=a ..b, opções);,

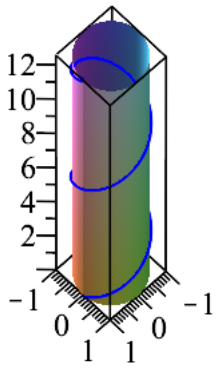
Note que a variação do parâmetro aqui está fora do colchete.

```
> restart; with( plots ) :
> alpha := t → [cos( t ), sin( t ), t ] : # a parametrização da curva (hélice)
> spacecurve( alpha( t ), t = 0 ..4 * Pi, scaling = constrained, color = blue, thickness = 2, orientation
      = [46, 26 ]); # o traço, estático
```

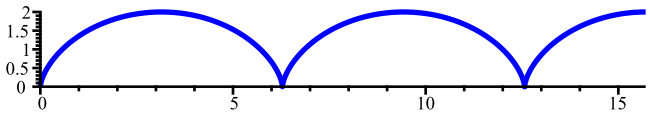


Animando a trajetória, com um fundo fixo:

```
> cilindro := plot3d([cos(u), sin(u), v], u = 0..2*Pi, v = 0..4*Pi, style = patchnogrid) :

  N := 10 :
  for i from 0 to N do
  s := i/N;
  quadro[i] := display({cilindro, spacecurve(alpha(t*s), t = 0..4*Pi, scaling = constrained,
      color = blue, thickness = 4, numpoints = 10*(i + 1), orientation = [46, 26])}) :
  od:
  display([seq(quadro[i], i = 0..N)], insequence = true);
```




```
> restart; with(plots) :
> plot( [t-sin(t), 1-cos(t), t=0..5*Pi], scaling=constrained, thickness=2, color=blue);
```

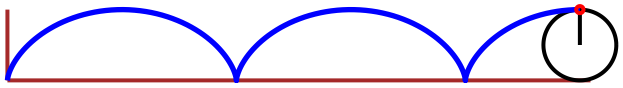


Vamos agora criar uma animação, para enxergar essa trajetória.

```
> restart; with(plots) : with(plottools) :
```

```
x := t → t - sin(t) :
y := t → 1 - cos(t) : # coordenadas do ponto
cx := t → t :
cy := t → 1 : # coordenadas do centro da circunferência
eixos := plot( {[[0, 0], [16, 0]], [[0, 0], [0, 2]]}, color = brown) :
N := 10 : # número de quadros da animação
t := 0 : passo := 5 * Pi / N :
for i from 0 to N do
ponto := circle( [x(t), y(t)], .1, color = red) :
curva := plot( [x(s), y(s), s = 0 ..t], color = blue, thickness = 2 ) :
roda := circle( [cx(t), cy(t)], 1, color = black) :
raio := plot( [[cx(t), cy(t)], [x(t), y(t)]], color = black) :
quadro[i] := display( [eixos, roda, raio, curva, ponto], scaling = constrained) :
t := t + passo :
od:
display( seq(quadro[i], i = 0 ..N ), insequence = true, title = `Ciclóide`, axes = none, scaling
    = constrained);
```

Ciclóide

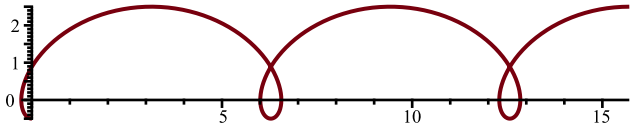


Voltar para ciclóide

Hiperciclóide

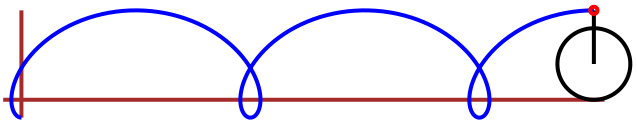
Vamos agora ver o que ocorre se o ponto estivesse na borda de uma roldana que se desloca sobre o eixo Ox, isto é, o raio ligando o ponto ao centro é maior que o raio interno da roldana.

```
> restart; with(plots) :
x := t→ t − 1.5 * sin(t) : y := t → 1 - 1.5 * cos(t) :
plot( [x(t), y(t), t = 0 ..5 * Pi], scaling = constrained);
```



Vamos animá-la, como anteriormente:

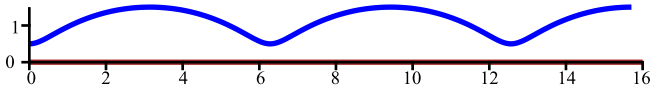
```
> restart; with(plots) : with(plottools) :
  x := t →t - 1.5 * sin(t) : y := t → 1 - 1.5 * cos(t) : # coordenadas do ponto
  cx := t → t : cy := t→ 1 : # coordenadas do centro da circunferência
 eixos := plot( {[[-.5, 0], [16, 0]], [[0,-.5], [0, 2.5]]}, color = brown) :
 N := 10 : # número de quadros da animação
  t := 0 : passo := 5 * Pi / N :
  for i from 0 to N do
 ponto := circle( [x(t), y(t)], .1, color = red) :
 curva := plot( [x(s), y(s), s = 0 ..t], color = blue) :
  roda := circle( [cx(t), cy(t)], 1, color = black) :
 raio := plot( [[cx(t), cy(t)], [x(t), y(t)]], color = black) :
  quadro[i] := display( [eixos, roda, raio, curva, ponto], scaling = constrained) :
  t := t + passo :
  od:
 display( seq(quadro[i], i = 0 ..N), insequence = true, title = `Hiperciclóide`, axes = none, scaling
      = constrained);
```



Hipociclóide

E se o ponto estivesse no interior da roda que se desloca sobre o eixo Ox?

```
> restart; with(plots) :
  x := t→ t − .5 * sin(t) : y := t → 1 − .5 * cos(t) :
  plot( [[x(t), y(t), t = 0 ..5 * Pi], [t, 0, t = 0 ..16]], scaling = constrained, color = [blue, brown],
        thickness = 2, tickmarks = [8, 3]);
```




```
> restart; with(plots) : with(plottools) :
  x := t → t − .5 * sin(t) :
```

```
y := t → 1 - .5 * cos(t) : # coordenadas do ponto
 cx := t → t :
cy := t → 1 : # coordenadas do centro da circunferência
eixos := plot( {[[0, 0], [16, 0]], [[0, 0], [0, 2]]}, color = brown) :
 N := 10 : # número de quadros da animação
 t := 0 : passo := 5 * Pi / N :
 for i from 0 to N do
 ponto := circle( [x(t), y(t)], .1, color = red) :
curva := plot( [x(s), y(s), s = 0 ..t], color = blue, thickness = 2 ) :
 roda := circle( [cx(t), cy(t)], 1, color = black) :
raio := plot( [[cx(t), cy(t)], [x(t), y(t)]], color = black) :
quadro[i] := display( [eixos, roda, raio, curva, ponto], scaling = constrained) :
t := t + passo :
od:
display( seq(quadro[i], i = 0 ..N), insequence = true, title = `Hipociclóide`, axes = none, scaling
      = constrained);
```

Hipociclóide