

UNIVERSIDADE FEDERAL DO RIO GRANDE DO SUL
INSTITUTO DE MATEMÁTICA
DEPARTAMENTO DE MATEMÁTICA PURA E APLICADA
Disciplina: MAT01168 -Matemática Aplicada -Semestre Letivo 2008/2
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SÉRIES DE FOURIER UTILIZANDO O MAPLE

Componente Par e Ímpar de uma Função

Toda função que não é par, nem é ímpar em seu domínio de definição, pode ser decomposta em sua parte par e sua parte ímpar, ou seja:

$$f(t) = f_{par}(t) + f_{impar}(t) ,$$

sendo estas partes definidas como

$$f_{par} := \frac{1f(t)}{2} + \frac{1f(-t)}{2} \quad e \quad f_{impar} := \frac{1f(t)}{2} - \frac{1f(-t)}{2}$$

EXEMPLOS

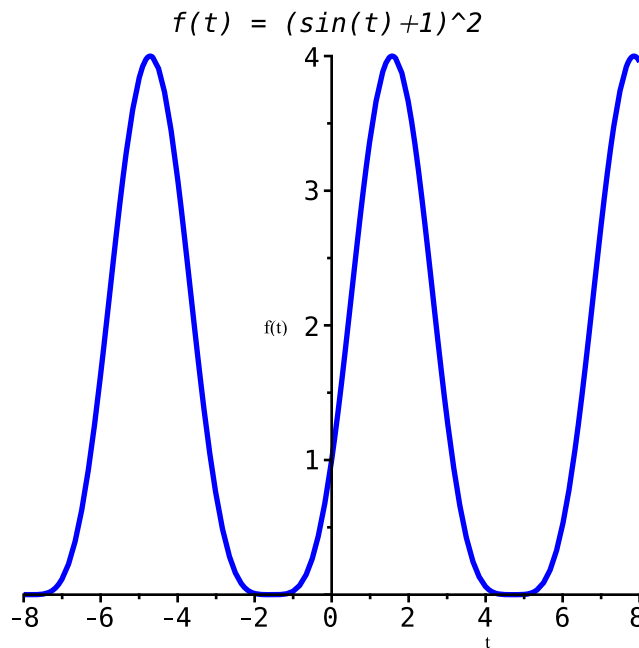
```
> restart:with(plots):with(plottools):  
Warning, the name changecoords has been redefined  
  
Warning, the name arrow has been redefined
```

1) A função $f(t) = (\sin(t)+1)^2$ não é par, nem é ímpar.

```
> f:=(sin(t)+1)^2;
```

$$f := (\sin(t) + 1)^2 \tag{1.1}$$

```
> gf:=plot(f,t=-8..8,color=blue,thickness=2,titlefont=[COURIER,  
DEFAULT,16],labels=["t", "f(t)],labelfont=[COURIER,DEFAULT,16],  
axesfont=[COURIER,DEFAULT,16],title=`f(t) = (sin(t)+1)^2` ):gf;
```



Vamos determinar a sua parte par:

```
> f1:=subs(t=-t,f);
```

$$f1 := (\sin(-t) + 1)^2 \quad (1.2)$$

```
> fpar:=(1/2)*(f+f1);
```

$$fpar := \frac{1}{2} (\sin(t) + 1)^2 + \frac{1}{2} (-\sin(t) + 1)^2 \quad (1.3)$$

```
> Fpar:=expand(fpar,trig);
```

$$Fpar := \sin(t)^2 + 1 \quad (1.4)$$

Agora, a sua parte ímpar:

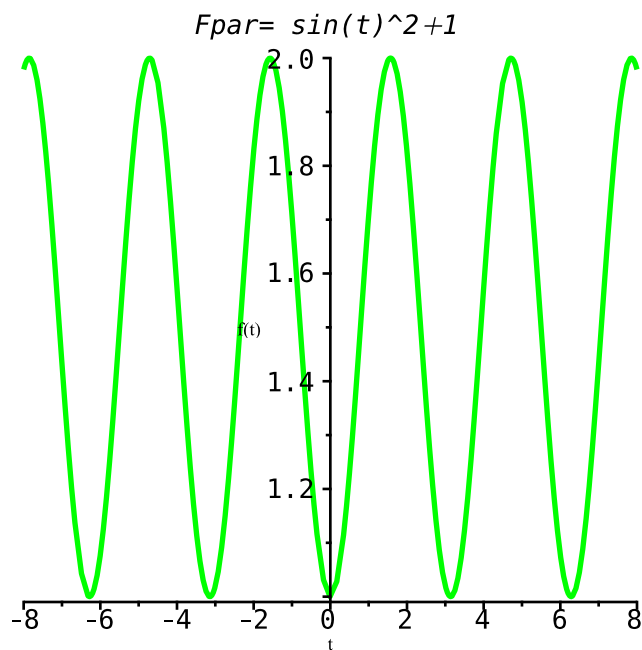
```
> fim:=(1/2)*(f-f1);
```

$$fim := \frac{1}{2} (\sin(t) + 1)^2 - \frac{1}{2} (-\sin(t) + 1)^2 \quad (1.5)$$

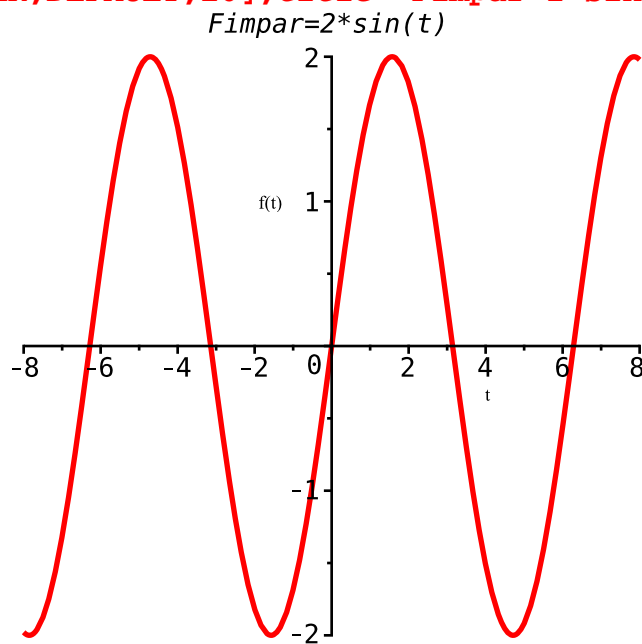
```
> Fimp:=expand(% ,trig);
```

$$Fimp := 2 \sin(t) \quad (1.6)$$

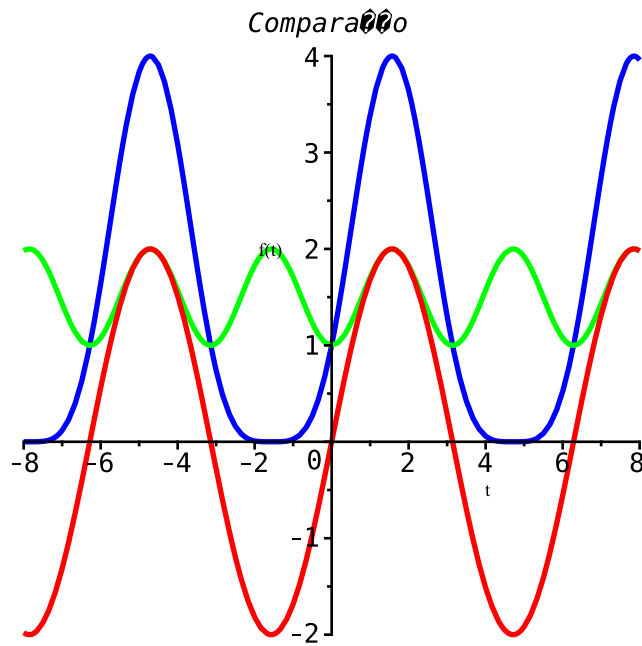
```
> gp:=plot(Fpar,t=-8..8,color=green,thickness=2,titlefont=[COURIER,
DEFAULT,16],labels=["t","f(t)"],labelfont=[COURIER,DEFAULT,16],
axesfont=[COURIER,DEFAULT,16],title=`Fpar= sin(t)^2+1` ):gp;
```



```
> gi:=plot(Fimp,t=-8..8,color=red,thickness=2,titlefont=[COURIER,
DEFAULT,16],labels=["t","f(t)"],labelfont=[COURIER,DEFAULT,16],
axesfont=[COURIER,DEFAULT,16],title=`Fimpar=2*sin(t)` ):gi;
```



```
> display({gf,gp,gi},title=`Comparação`);
```



```
> Fsoma:=Fpar+Fimp;
```

$$Fsoma := \sin(t)^2 + 2 \sin(t) + 1$$

(1.7)

```
2)f(t)=(t+1)/(t-1);
```

```
> restart:with(plots):with(plottools):
```

```
Warning, the name changecoords has been redefined
```

```
Warning, the name arrow has been redefined
```

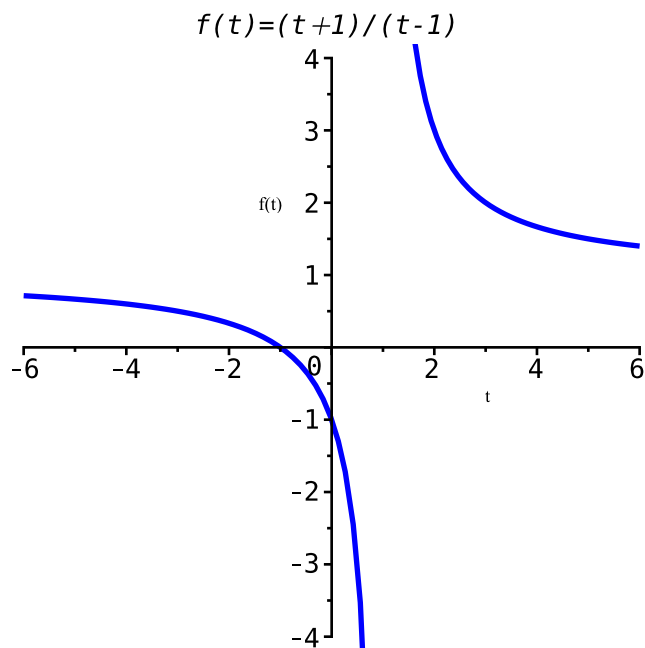
```
> f:=(t+1)/(t-1);
```

$$f := \frac{t+1}{t-1}$$

(1.8)

Gráfico da função dada

```
> gf:=plot(f,t=-6..6,-4..4,color=blue,thickness=2,discont=true,
titlefont=[COURIER,DEFAULT,16],labels=["t","f(t)],labelfont=
[COURIER,DEFAULT,16],axesfont=[COURIER,DEFAULT,16],title=`f(t)=
(t+1)/(t-1)`):gf;
```



Vamos determinar a sua parte par:

```
> f1:=subs(t=-t,f);
```

$$f1 := \frac{-t+1}{-t-1} \quad (1.9)$$

```
> fpar:=(1/2)*(f+f1);
```

$$fpar := \frac{t+1}{2(t-1)} + \frac{-t+1}{2(-t-1)} \quad (1.10)$$

```
> Fpar:=radsimp(fpar);
```

$$Fpar := \frac{t^2+1}{(t-1)(t+1)} \quad (1.11)$$

Agora, a sua parte ímpar:

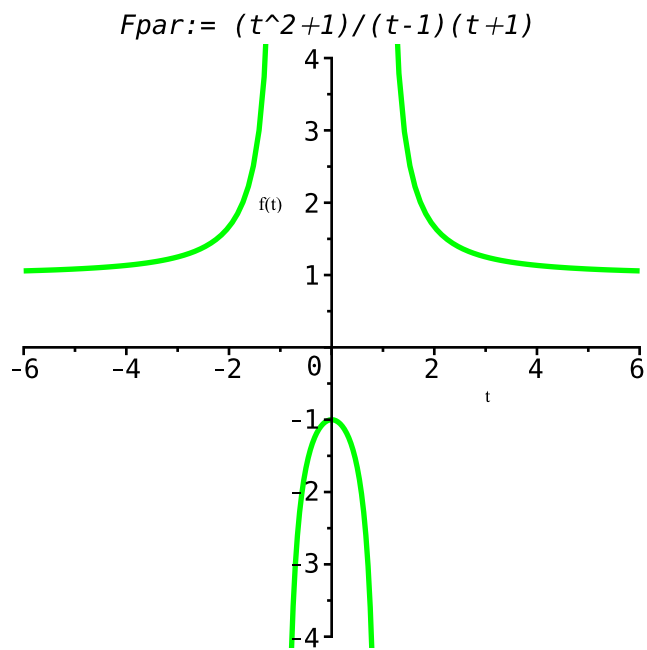
```
> fim:=(1/2)*(f-f1);
```

$$fim := \frac{t+1}{2(t-1)} - \frac{-t+1}{2(-t-1)} \quad (1.12)$$

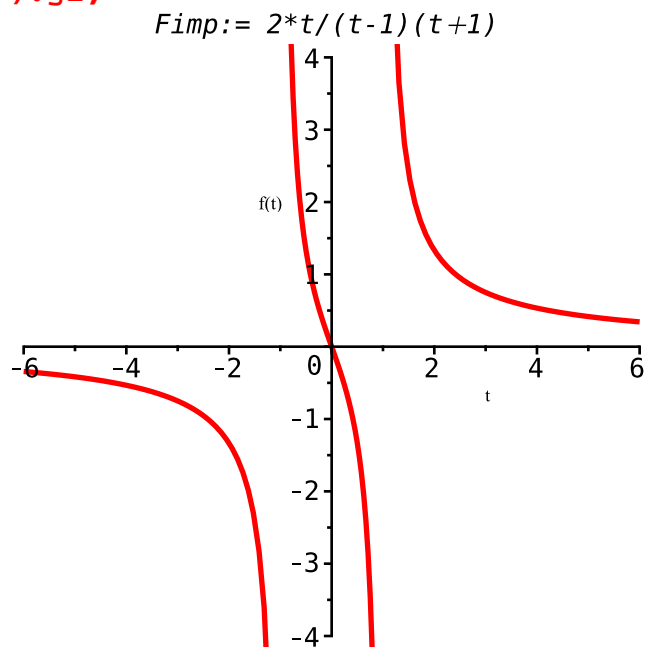
```
> Fimp:=radsimp(fim);
```

$$Fimp := \frac{2t}{(t-1)(t+1)} \quad (1.13)$$

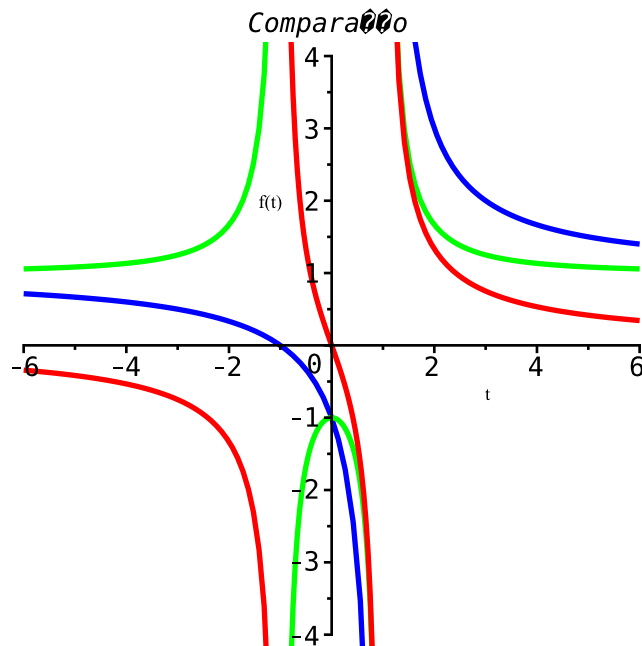
```
> gp:=plot(Fpar,t=-6..6,-4..4,color=green,thickness=2,discont=true,
titlefont=[COURIER,DEFAULT,16],labels=["t","f(t)],labelfont=
[COURIER,DEFAULT,16],axesfont=[COURIER,DEFAULT,16],title=`Fpar:=
(t^2+1)/(t-1)(t+1)`):gp;
```



```
> gi:=plot(Fimp,t=-6..6,-4..4,discont=true,color=red,thickness=2,
titlefont=[COURIER,DEFAULT,16],labels=["t","f(t)],labelfont=
[COURIER,DEFAULT,16],axesfont=[COURIER,DEFAULT,16],title=`Fimp:=
2*t/(t-1)(t+1)`):gi;
```



```
> display({gf,gp,gi},title=`Comparação`);
```



```
> Fsoma:=radsimp(Fpar+Fimp);
```

$$Fsoma := \frac{t + 1}{t - 1}$$

(1.14)

3) $f(t) = e^t$

```
> restart:with(plots):with(plottools):
```

```
Warning, the name changecoords has been redefined
```

```
Warning, the name arrow has been redefined
```

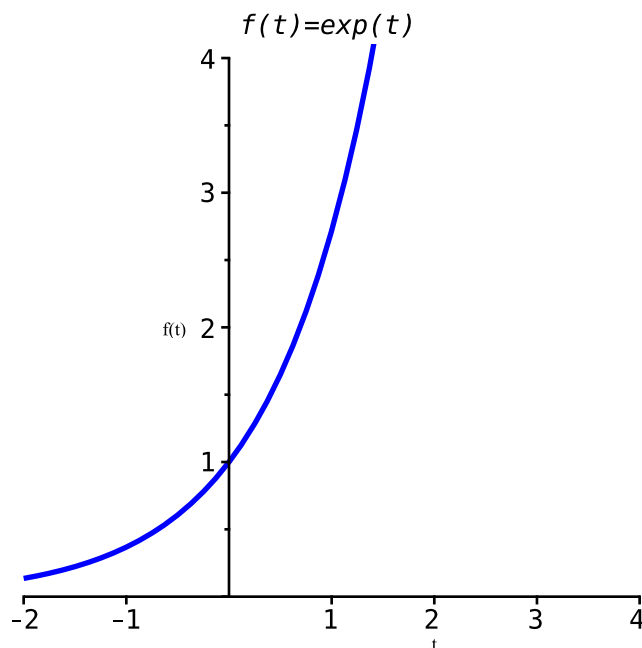
```
> f:=exp(t);
```

$$f := e^t$$

(1.15)

Gráfico da função dada

```
> gf:=plot(f,t=-2..4,0..4,color=blue,thickness=2,xtickmarks=[-4,-3,-2,-1,1,2,3,4],title=`f(t)=exp(t)`,titlefont=[COURIER,DEFAULT,16],labels=["t","f(t)],labelfont=[COURIER,DEFAULT,16],axesfont=[COURIER,DEFAULT,16]):gf;
```



Vamos determinar a sua parte par:

```
> f1:=subs(t=-t,f);
```

$$f1 := e^{-t} \quad (1.16)$$

```
> fpar:=(1/2)*(f+f1);Fpar:=convert(fpar,trig);
```

$$fpar := \frac{1}{2} e^t + \frac{1}{2} e^{-t} \quad (1.17)$$

$$Fpar := \cosh(t)$$

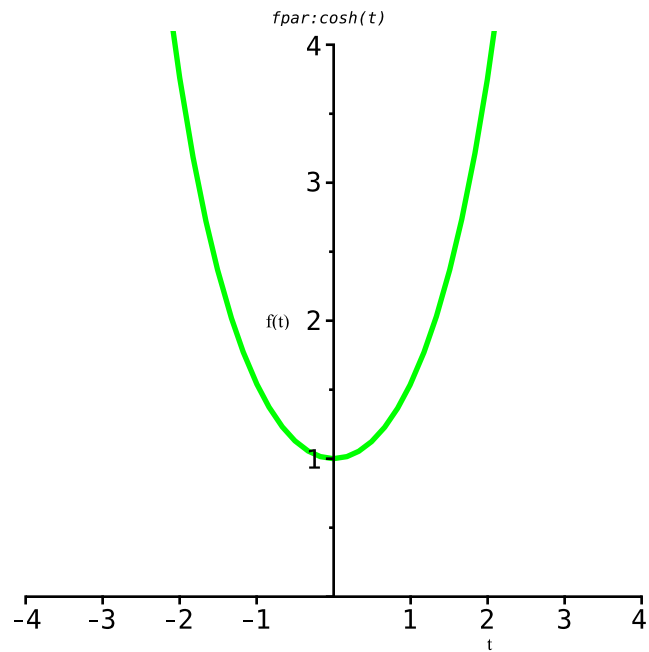
Agora, a sua parte ímpar:

```
> fimp:=(1/2)*(f-f1);Fimp:=convert(fimp,trig);
```

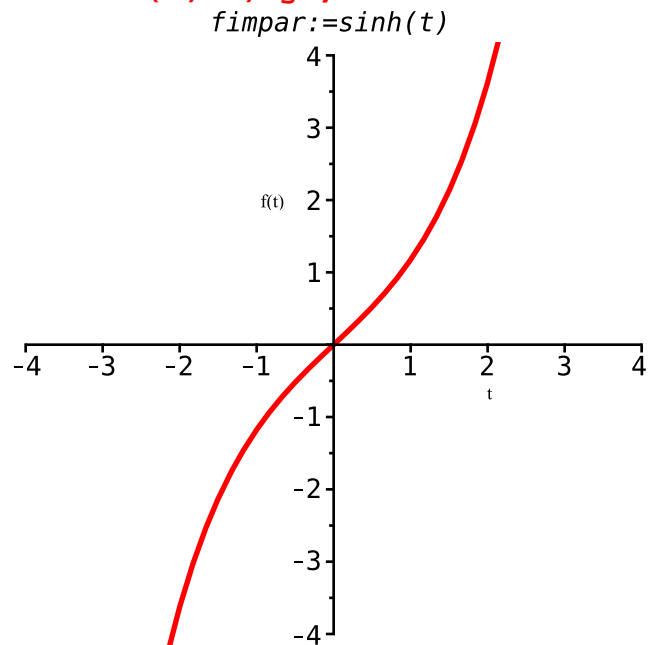
$$fimp := \frac{1}{2} e^t - \frac{1}{2} e^{-t} \quad (1.18)$$

$$Fimp := \sinh(t)$$

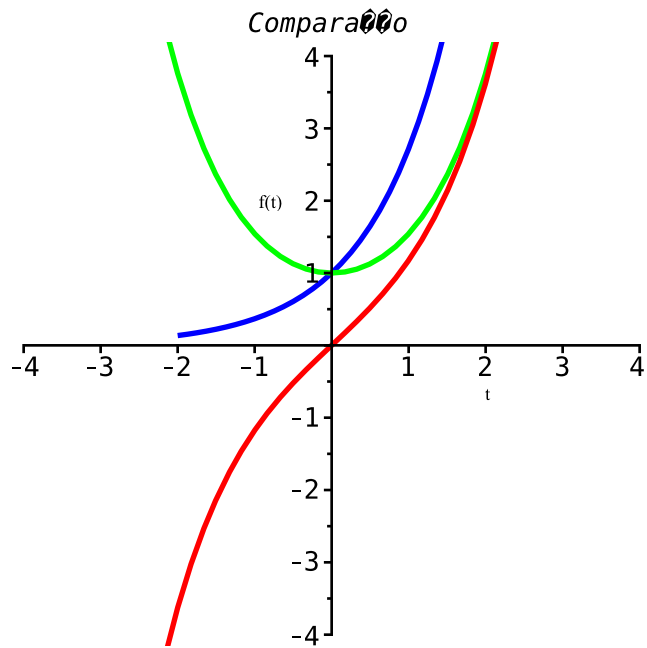
```
> gp:=plot(Fpar,t=-4..4,0..4,color=green,xtickmarks=[-4,-3,-2,-1,1,2,3,4],thickness=2,titlefont=[COURIER,DEFAULT,10],labels=["t","f(t)"],labelfont=[COURIER,DEFAULT,16],axesfont=[COURIER,DEFAULT,16],title=`fpar:cosh(t)`):gp;
```

```
> gi:=plot(Fimp,t=-4..4,-4..4,color=red,xtickmarks=[-4,-3,-2,-1,1,2,3,4],thickness=2,titlefont=[COURIER,DEFAULT,16],labels=["t","f(t)"],labelfont=[COURIER,DEFAULT,16],axesfont=[COURIER,DEFAULT,16],title=`fimpar:=sinh(t)`):gi;
```



```
> display({gf,gp,gi},title=`Comparação`);
```



```
> Fsoma:=radsimp(fpar+fimp);
```

$$Fsoma := e^t$$

(1.19)

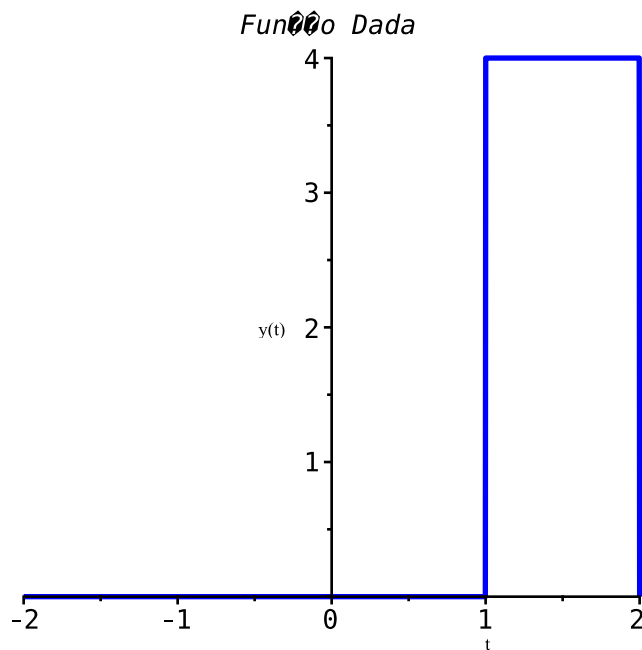
4 Encontre a parte par e ímpar do sinal : $y(t) := \begin{cases} 0 & -t < 0 \text{ and } t < 1 \\ 4 & -t < -1 \text{ and } t < 2 \end{cases}$

```
> y(t):=piecewise(t>0 and t<1,0,t>1 and t<2,4);
```

$$y(t) := \begin{cases} 0 & -t < 0 \text{ and } t < 1 \\ 4 & -t < -1 \text{ and } t < 2 \end{cases}$$

(1.20)

```
> plot(y(t),t=-2..2,thickness=2,color=blue,titlefont=[COURIER,
DEFAULT,16],labels=["t","y(t)],labelfont=[COURIER,DEFAULT,16],
axesfont=[COURIER,DEFAULT,16],title=`Função Dada`);
```



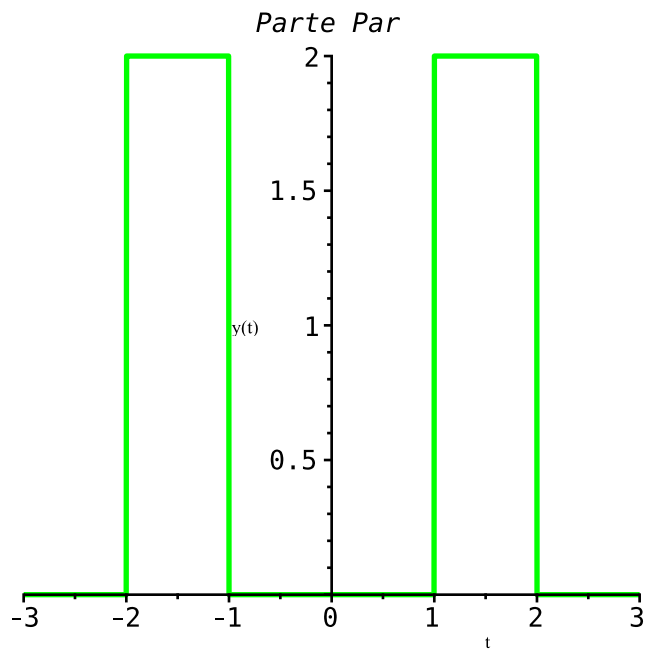
```
> y(-t):=piecewise(-t<0 and -t<1,0,-t>1 and -t<2,4);
```

$$y(-t) := \begin{cases} 0 & -t < 0 \text{ and } -t < 1 \\ 4 & -t > 1 \text{ and } -t < 2 \end{cases} \quad (1.21)$$

```
> ypar:=simplify((1/2)*(y(t)+y(-t)));
```

$$ypar := \begin{cases} 0 & t \leq -2 \\ 2 & -2 < t < -1 \\ 0 & -1 \leq t \leq 1 \\ 2 & 1 < t < 2 \\ 0 & 2 \leq t \end{cases} \quad (1.22)$$

```
> plot(ypar,t=-3..3,thickness=2,color=green,titlefont=[COURIER,
DEFAULT,16],labels=["t","y(t)"],labelfont=[COURIER,DEFAULT,16],
axesfont=[COURIER,DEFAULT,16],title=`Parte Par`);
```

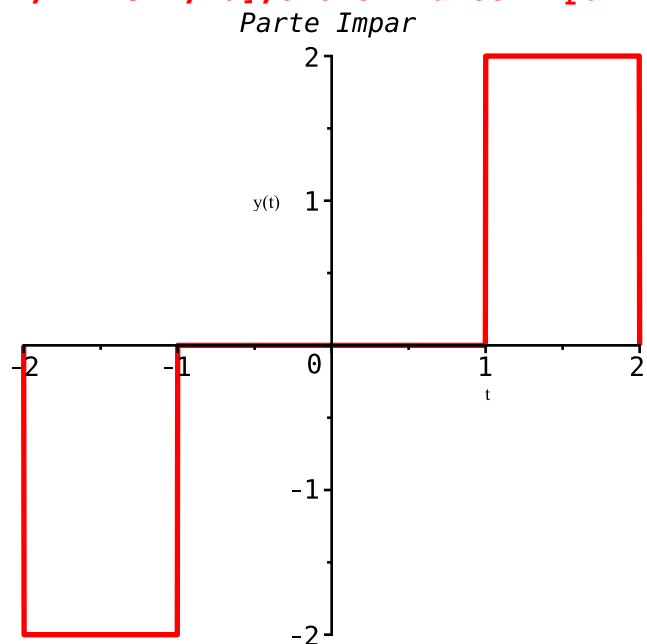


```
> yimp:=simplify((1/2)*(y(t)-y(-t)));
```

$$yimp := \begin{cases} 0 & t \leq -2 \\ -2 & -2 < t < -1 \\ 0 & -1 \leq t \leq 1 \\ 2 & 1 < t < 2 \\ 0 & 2 \leq t \end{cases}$$

(1.23)

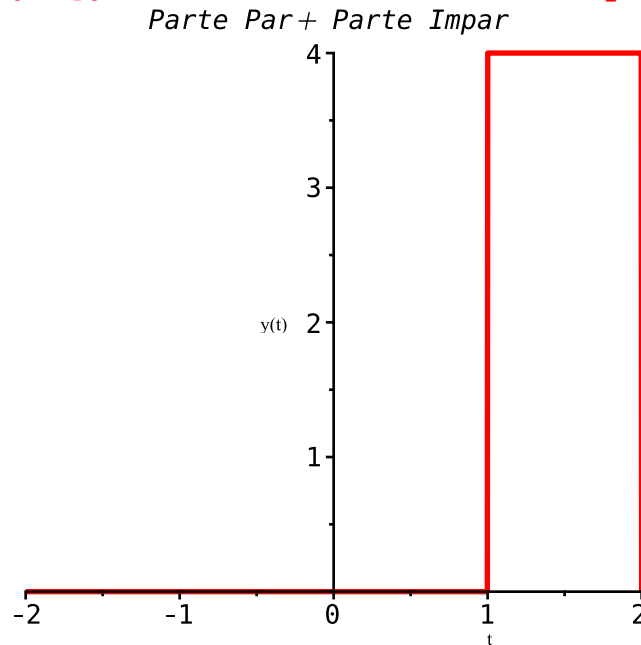
```
> plot(yimp,t=-2..2,color=red,thickness=2,titlefont=[COURIER,
DEFAULT,16],labels=["t","y(t)"],labelfont=[COURIER,DEFAULT,16],
axesfont=[COURIER,DEFAULT,16],title=`Parte Impar`);
```



```
> ysoma:=simplify(ypar+yimp);
```

$$ysoma := \begin{cases} 0 & t \leq 1 \\ 4 & t < 2 \\ 0 & 2 \leq t \end{cases} \quad (1.24)$$

```
> plot(ysoma,t=-2..2,thickness=2,titlefont=[COURIER,DEFAULT,16],
labels=["t","y(t)],labelfont=[COURIER,DEFAULT,16],axesfont=
[COURIER,DEFAULT,16],title=`Parte Par+ Parte Impar`);
```



5) Encontre a parte par e ímpar do sinal : $y(t) := \begin{cases} 4t + 4 & -t < 1 \text{ and } t < 0 \\ 4 - 2t & -t < 0 \text{ and } t < 2 \end{cases}$

```
> restart:with(plots):with(plottools):
```

Warning, the name changecoords has been redefined

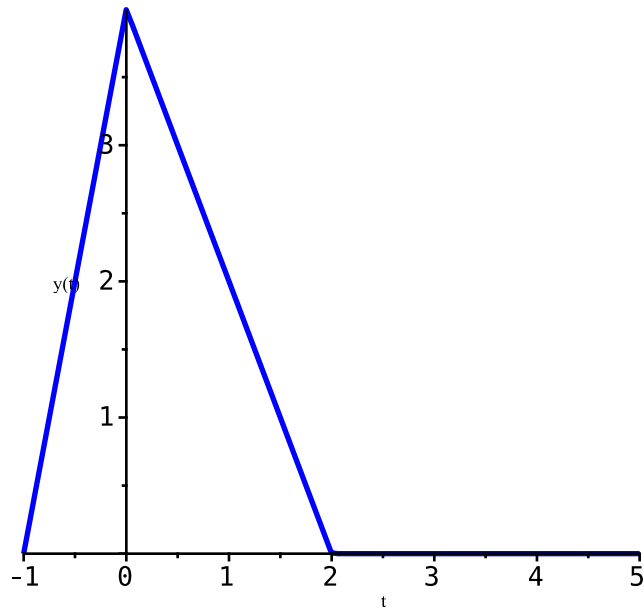
Warning, the name arrow has been redefined

```
> y(t):=piecewise(t>-1 and t<0,4*t+4,t>0 and t<2,4-2*t);
```

$$y(t) := \begin{cases} 4t + 4 & -t < 1 \text{ and } t < 0 \\ 4 - 2t & -t < 0 \text{ and } t < 2 \end{cases} \quad (1.25)$$

```
> plot(y(t),t=-1..5,thickness=2,color=blue,titlefont=[COURIER,
DEFAULT,16],labels=["t","y(t)],labelfont=[COURIER,DEFAULT,16],
axesfont=[COURIER,DEFAULT,16],title=`Função Dada`);
```

Função Dada



```
> y(-t):=subs(t=-t,y(t));
```

$$y(-t) := \begin{cases} -4t + 4 & t < 1 \text{ and } -t < 0 \\ 4 + 2t & t < 0 \text{ and } -t < 2 \end{cases}$$

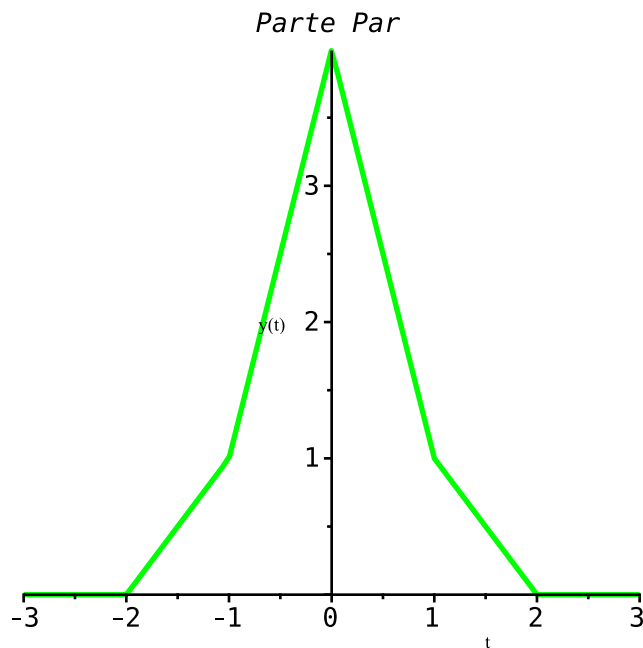
(1.26)

```
> ypar:=simplify((1/2)*(y(t)+y(-t)));
```

$$ypar := \begin{cases} 0 & t \leq -2 \\ 2 + t & t \leq -1 \\ 3t + 4 & t < 0 \\ 0 & t = 0 \\ 4 - 3t & t < 1 \\ 2 - t & t < 2 \\ 0 & 2 \leq t \end{cases}$$

(1.27)

```
> plot(ypar,t=-3..3,thickness=2,color=green,titlefont=[COURIER,
DEFAULT,16],labels=["t","y(t)],labelfont=[COURIER,DEFAULT,16],
axesfont=[COURIER,DEFAULT,16],title=`Parte Par`);
```

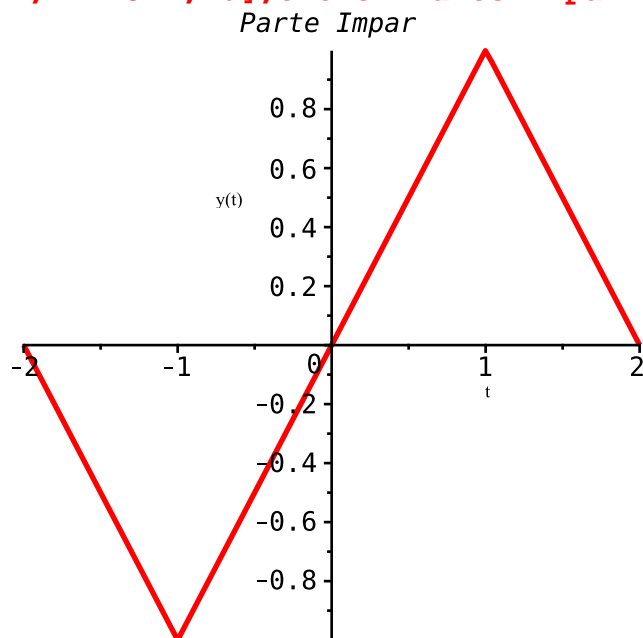


```
> yimp:=simplify((1/2)*(y(t)-y(-t)));
```

$$yimp := \begin{cases} 0 & t \leq -2 \\ -t-2 & -2 < t \leq -1 \\ t & -1 < t < 1 \\ 2-t & 1 \leq t < 2 \\ 0 & 2 \leq t \end{cases}$$

(1.28)

```
> plot(yimp,t=-2..2,color=red,thickness=2,titlefont=[COURIER,
DEFAULT,16],labels=["t","y(t)"],labelfont=[COURIER,DEFAULT,16],
axesfont=[COURIER,DEFAULT,16],title=`Parte Impar`);
```



```
> ysoma:=simplify(ypar+yimp);
```

$$ysoma := \begin{cases} 0 & t \leq -1 \\ 4t + 4 & -1 < t < 0 \\ 0 & t = 0 \\ 4 - 2t & 0 < t < 2 \\ 0 & 2 \leq t \end{cases} \quad (1.29)$$

```
> plot(ysoma,t=-2..2,thickness=2,titlefont=[COURIER,DEFAULT,16],
labels=["t","y(t)],labelfont=[COURIER,DEFAULT,16],axesfont=
[COURIER,DEFAULT,16],title=`Parte Par + Parte Impar`);
      Parte Par + Parte Impar
```

